

FACULTY OF ENGINEERING

B.E. III Semester (CBCS) (Except I.T) (Back'og) Examination, December 2019

Subject: Engineering Mathematics- III

Time: 3 Hours

Max. Marks:75

Note: Answer all questions from Part-A & any five questions from Part-B

PART - A (25 Marks)

1. If $f(z) = u(x, y) + iv(x, y)$ is analytic function, then prove that $u(x, y)$ and $v(x, y)$ are harmonic functions. (3)
2. Evaluate $\oint_C \frac{\sin^2 z}{z - \frac{\pi}{6}} dz$, where C is the circle $|z| = 1$. (2)
3. Determine the pole of the function $f(z) = \frac{z^2}{(z-1)^2(z+2)}$ and find the residue at each point. (2)
4. Expand $f(z) = \frac{1}{(z-1)(z-2)}$ in the region $|z| < 1$. (3)
5. Define Dirichlet's conditions for the existence of Fourier series of a function $f(x)$. (2)
6. Find the half range sine series of $f(x) = x, x \in (0, \pi)$. (3)
7. Form the partial differential equation by eliminating arbitrary functions from $Z = f(x+at) + g(x-at)$. (3)
8. Obtain complete solution of $pq + p + q = 0$. (2)
9. Solve $3\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0, u(x, 0) = 4e^{-x}$. (3)
10. Classify the partial differential equation $\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 u}{\partial x \partial y} + 4\frac{\partial^2 z}{\partial y^2} = 0$. (2)

PART - B (5 x 10 = 50 Marks)

11. a) Find the analytic function $f(z) = u + iv$, if $u - v = (x - y)(x^2 + 4xy + y^2)$. (6)
- b) Evaluate $\oint_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$ where C is the circle $|z| = 3$. (4)
12. a) Find Laurent's expansion of $f(z) = \frac{7z-2}{z(z+1)(z-2)}$ in the region $1 < |z+1| < 3$. (4)
- b) Evaluate $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)(x^2+4)} dx$. (6)
13. a) $f(x) = |\cos x|$, expand $f(x)$ as a Fourier series in the interval $(-\pi, \pi)$. (10)
14. a) Solve $(x^2 - y^2 - z^2)p + 2xyq = 2xz$. (5)
- b) Solve $2z + p^2 + qy + 2y^2 = 0$ by using Charpit's method. (5)

15. A homogeneous rod of conducting material of length 100 cm has its ends kept at zero temperature and the temperature initially is $u(x, 0) = \begin{cases} x, & 0 \leq x \leq 50 \\ 100 - x & 50 \leq x \leq 100 \end{cases}$ (10)

Find the temperature $u(x, t)$ at any time.

16. Find the bilinear transformation which maps the points $z = 1, i, -1$ onto the points $w = i, 0, -i$. (4)

16. Evaluate $\oint_C \frac{z}{(z-1)(z-2)^2} dz$ where C is the circle $|z-2| = \frac{1}{2}$. (6)

17. a) Solve $\frac{\partial^3 z}{\partial x^3} - 2 \frac{\partial^3 z}{\partial x^2 \partial y} = 2e^{2x} + 3x^2 y$. (5)

b) Find the complete solution of $z^2(p^2 + q^2) = x^2 + y^2$. (5)

downloaded from
StudentSuvidha.com